BT-MRE-I & II(S)-05.16-0758 Reg. No. | | | | | | | | |

D

B.Tech. Degree I & II Semester Supplementary Examination in Marine Engineering May 2016

MRE 101 ENGINEERING MATHEMATICS I

Time: 3 Hours Maximum Marks: 100

(5 × 20 = 100)

I. (a) Show that the function $f(x)=x\sin\frac{1}{x},\ x\neq 0$ is continuous at $x=0$ but not differentiable there.

(b) Evaluate $\lim\limits_{\theta\to 0}\dfrac{\theta-\sin\theta}{\sin\theta(1-\cos\theta)}$.

(c) State Rolle's theorem verify it for $f(x)=e^x(\sin x-\cos x)$ in $(\pi/4,\ 5\pi/4)$.

OR

II. (a) Find the radius of curvature at any point $(at^2, 2at)$ of the parabola.

(b) Find the n[th] derivative of $e^x\cos x\cos 2x$.

(c) If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$.

III. (a) If $u=F(x-y,\ y-z,\ z-x)$ prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$.

(b) If $u=\sin^{-1}\dfrac{x^2+y^2}{\sqrt{x}+\sqrt{y}}$ prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{3}{2}\tan u$.

(c) The radius of a circle is found to be 100 cm. Find the relative error in the area of the circle due to an error of 1 mm in measuring the radius.

OR

IV. (a) If $x^3+3x^2y+6xy^2+y^3=1$, find $\dfrac{dy}{dx}$.

(b) Find the maximum and minimum distances from the origin to the curve $5x^2+6xy+5y^2-8=0$.

(c) If $u=\dfrac{xy}{z},\ v=\dfrac{yz}{x},\ w=\dfrac{zx}{y}$ find the Jacobians $J=\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ and $J'=\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. Verify that $JJ'=1$.

V. (a) Show that the locus of midpoints of chords of the parabole $y^2=4ax$, which subtend a right angle at the vertex is $y^2=2a(x-4a)$.

(b) Find the equation of the tangent and normal at $(a\cos\theta, b\sin\theta)$ of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

(c) Derive the standard equation of hyperbola.

OR

(P.T.O.)

VI. (a) For the hyperbola $9x^2 - 16y^2 = 144$, find (i) the semi axes (ii) the foci (iii) the eccentricity (iv) the Latus Rectum.

(b) Prove that the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is the curve $y^2 = a(x - 3a)$.

(c) Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

VII. (a) Find the area included between the curve $y^2(2a - x) = x^3$ and its asymptote.

(b) Find the volume obtained by revolving an arch of the cycloid $x = a(\theta + \sin\theta)$ $y = a(1 + \cos\theta)$.

(c) Evaluate $\int_0^1 \int_y^{y+1} x^2 y \, dx \, dy$.

OR

VIII. (a) Describe the region of integration and evaluate $\int_0^1 \int_x^{2x} (1 + x^2 + y^2) dx \, dy$.

(b) $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$.

IX. (a) Prove that the area of the triangle whose vertices are A, B, C is $\frac{1}{2}[B \times C + C \times A + A \times B]$.

(b) Show that the points $-6I + 3J + 2K$, $3I - 2J + 4K$, $5I + 7J + 3K$ and $-13I + 17J - K$ are coplanar.

(c) Prove that $(\vec{B} \times \vec{C}).(\vec{A} \times \vec{D}) + (\vec{C} \times \vec{A}).(\vec{B} \times \vec{D}) + (\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = 0$.

OR

X. (a) A vector field is given by $\vec{A} = (x^2 + xy)\hat{i} + (y^2 + x^2 y)\hat{j}$. Show that the field is irrotational and find the scalar potential.

(b) Prove that $div(\vec{A} + \vec{B}) = div\vec{A} + div\vec{B}$.

(c) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

***